NOTATION

$p = P^{0.5}$ is the pressure, Nm^{-2};
p_e, p_0 are the initial and atmospheric pressures, Nm^{-2};
μ is the concentration, kg/kg;
m_0 is the porosity;
v_0, v are the initial air speed and speed relative to material, m/sec;
L is the pipeline length;
H is the depth of bed in tube;
x, z are the coordinates;
t is the time.

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REMOVAL OF A LIQUID FROM AN OPEN-CELL BODY BY FLUIDIZED POROUS PARTICLES 1.

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It is shown that an analytic description can be given for the elimination of a bound material from a porous body immersed in a fluidized bed composed of small porous particles; the equations for fluid transport in a porous space can be used.

Here we consider the elimination of a bound fluid from a porous ceramic semifinished product during preliminary thermal processing in a fluidized bed [1-3].

Pure-oxide ceramics are produced mainly in semifinished form by hot pressing with a wax binding agent [4]. A major step in manufacturing such components that precedes the final firing is to eliminate the binding agent, which may be performed in a fluidized bed [5]. The process is operated at temperatures below the onset of evaporation of the binding agent. In that case, the vapor transport can be essentially neglected, so the internal mass transfer in the porous system occurs only in the liquid state.

A difference of our treatment from previous ones [6, 7] is that we derive solutions from the liquid-transport equations for film motion [8, 9] in a model porous medium. The model is a system consisting of capillaries of radii R_1 and R_2 interconnected throughout their length. This corresponds to the actual porous structure of numerous ceramic materials, in particular in that it fits the bimodal pore-size distribution [10, 11]. If the body is immersed in a fluidized bed at a temperature well below the evaporation point of the liquid, the only cause of external mass transport is liquid loss to the particles on collision with the surface (Fig. 1).

The capillary potential of a porous particle $P_p = 2\sigma \cos\theta / r$ is less than the potential of the liquid at the surface of the body $P_0(\tau)$, so the liquid is drawn into the capillaries of the particles when the latter are near the surface.

The following is the equation for the momentum change for the liquid in a particle capillary:

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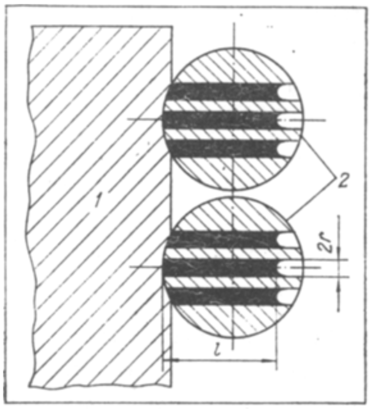

Fig. 1

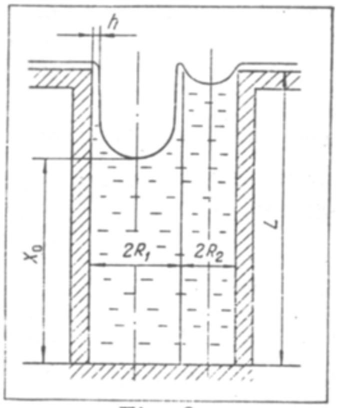

Fig. 2

Fig. 1. Interaction of a body with porous particles: 1) porous body; 2) particles.

Fig. 2. Model for a porous body and scheme for calculating the first stage.

$$\frac{d(mV)}{d\tau} = \left[\frac{2\sigma\cos\theta}{r} - P_0(\tau)\right]\pi r^2 - F_f(l, V). \tag{1}$$

The flow of the liquid in the capillary in the particle can be represented via Poiseuille's law as

$$q = \frac{\pi r^4}{8\eta}\frac{\Delta P}{l} = \pi r^2 V, \tag{2}$$

and therefore the pressure difference over the length of the capillary is

$$\Delta P = \frac{8\eta l V}{r^2}. \tag{3}$$

The frictional force is

$$F_f = \Delta P \pi r^2 = 8\eta l V. \tag{4}$$

Then (4) converts (1) to

$$\rho r^2 d\left(l, \frac{dl}{d\tau}\right) = r^2 (P_p - P_0) - 8\eta l \frac{dl}{d\tau}. \tag{5}$$

We get from (5) for $l \to 0$ that

$$\left(\frac{dl(0)}{d\tau}\right)^2 = \frac{P_p - P_0}{\rho}. \tag{6}$$

As $l(0) = 0$, we have from (5) and (6) that

$$l(\tau) = \left\{\frac{(P_p - P_0) r^2}{4\eta}\tau - \frac{(P_p - P_0)\rho r^4}{32\eta^2}\left[1 - \exp\left(-\frac{8\eta}{\rho r^2}\tau\right)\right]\right\}^{0.5}. \tag{7}$$

We now define the conditions under which $\exp[-(8\eta/\rho r^4)\tau]$ differs noticeably from zero; for wax (ρ = 900 kg/m^3, η = 0.45 Poise) and particles having capillaries of radii $r = 10^{-6}$ cm we get that this occurs for $\tau \ll 10^{-12}$ sec; however, the time of contact between the particles and the immersed body is [13] much greater, and in that case we get instead of (7) that

$$l(\tau) = \sqrt{\frac{(P_p - P_0) r^2}{8\eta}\tau}, \tag{8}$$

which corresponds to the standard condition for quasistationary flow.

From (8) we get the flow time for the case $l = d_p$, which shows that a porous particle is not completely impregnated with liquid during the contact with the body; therefore, the τ of (8) is taken in the subsequent calculations as the time of contact τ_c between the particle and the surface. The amount of liquid taken up by one particle during the time of contact is:

$$M = \pi r^2 \rho l N S \omega (P_0). \tag{9}$$

Here $\omega(P_0)$ is the probability that dry parts of the surface of the particle will contact capillaries in the porous body filled with liquid; we assume that the surface and volume fluid contents of the particle are equal to get that

$$\omega(P_0) = n_p n W_s (1 - W_p), \tag{10}$$

in which W_s is the surface fluid content of the body and W_p is the mean fluid content of the particles.

With account taken of (10), the final expression for isothermal flow of fluid, taken up by particles of the fluidized bed, will then be

$$i = M\bar{n} = \pi r^2 N S n_p n W_p (1 - W_p)(\tau_c/8\eta)^{0.5} \sqrt{P_p - P_0}\, \rho \bar{n},$$
$$\alpha = \rho S n_p^2 n \bar{n} (1 - f_0)(\tau_c/8\eta)^{0.5} r, \tag{11}$$

in which $\bar{n}$ and τ_c define the hydrodynamic conditions in the fluidized bed close to the bodies immersed in it [12, 13].

We now write the expression for the liquid flow in the capillaries, restricting consideration to the case where the difference in capillary pressures at the menisci of the small and large capillaries is sufficient to take up an amount of liquid such that the menisci in the narrow capillaries do not alter in position (Fig. 2). Therefore, the liquid is removed by the porous particles only from the surfaces of small capillaries. We neglect the loss of liquid from the film formed at the mouths of the large capillaries. We also assume that the capillary walls are completely wetted by the liquid, while the flow through the small capillaries can [14] be put as

$$j_2 = \frac{\rho R_2^2}{8\eta (L - X_0)} \Delta P. \tag{12}$$

The potential P_0 at the surface of the porous body varies from $P_{0i} = 0$ to that determined by the curvature of the meniscus in a narrow capillary, $P_{0f} = 2\sigma/R_1$.

The fluid flows in the wide capillaries as a film in response to the pressure difference Π [9]:

$$j_1 = \frac{2\rho h^3}{3\eta R_1} \cdot \frac{d\Pi}{dX}. \tag{13}$$

We assume that this wedging pressure has the following relationship to thickness [15] for nonpolar liquids:

$$\Pi = A/h^3, \tag{14}$$

and we express the pressure in the liquid film in terms of this pressure and the surface curvature as follows:

$$P = \frac{\sigma}{R_1} + \Pi(h) = \frac{\sigma}{R_1} + \frac{A}{h^3}. \tag{15}$$

From (15) with $P = P(X)$, $P_0 = P(0)$, and $P_1 = 2\sigma/R_1 = P(L)$

$$h = \frac{A^{1/3}}{[P(X) - \sigma/R_1]^{1/3}}. \tag{16}$$

Substitution of (16) into (13) and appropriate transformation give us the liquid flow rate in the wide capillaries as

$$j_1 = \frac{2\rho A}{3\eta R_1 (L - X_0)} \ln\left(\frac{P_0 R_1}{\sigma} - 1\right). \tag{17}$$

The total flows in the capillaries of radii R_1 and R_2 are put, respectively, as

$$J_1 = \frac{2\rho A F_1}{3\eta R_1 (L - X_0)} \ln\left(\frac{P_0 R_1}{\sigma} - 1\right), \tag{18}$$

$$J_2 = \frac{\rho R_2^2 F_2}{8\eta (L - X_0)} \left(P_0 - \frac{2\sigma}{R_1}\right). \tag{19}$$

The rate of removal of liquid from the body in the first stage is characterized by the condition $L \geq X \geq X_c$ and is

$$i = \frac{J_1 + J_2}{F} = \frac{2\rho A n_1}{3\eta R_1 (L - X_0)} \ln\left(\frac{P_0 R_1}{\sigma} - 1\right) + \frac{\rho R_2^2 n_2}{8\eta (L - X_0)} \left(P_0 - \frac{2\sigma}{R_1}\right).$$

All the liquid brought up to the surface is removed by colliding porous particles, so the flows given by (11) and (20) may be equated:

$$\frac{2\rho A n_1}{3\eta R_1 (L-X_0)} \ln\left(\frac{P_0 R_1}{\sigma}-1\right)+\frac{\rho R_2^2 n_2}{8\eta (L-X_0)}\left(P_0-\frac{2\sigma}{R_1}\right)=\alpha W_s\,(1-W_p)\sqrt{P_p-P_0}. \quad (21)$$

The following is the relationship between the rate of descent of the menisci in the broad capillaries and the surface potential P_0:

$$n_1\rho\,\frac{dX_0}{d\tau}=-\alpha W_s\,(1-W_p)\sqrt{P_p-P_0}. \quad (22)$$

The surface liquid content of the body in the first stage (neglecting the films in the broad capillaries) may be put as

$$W_s=n_1\frac{2h_0}{R_1}+n_2\sim n_2.$$

In what follows we assume that the number of particles in the mass-transfer volume is large, which means that the surface of the porous body always receives only dry particles; i.e., $W_p = 0$. In practice this means that the particles are continuously eliminated from the volume and replaced by dry particles.

Then (2) gives us the transport potential at the surface as

$$P_0=P_p-\frac{n_1^2\rho^2}{n_2^2\alpha^2}\left(\frac{dX_0}{d\tau}\right)^2. \quad (23)$$

We substitute (23) into (21) to get an equation relating the amount of liquid present in the porous system at a given time and the rate of descent of the menisci in the wide capillaries:

$$L-X_0=\frac{2A}{3\eta R_1 (dX_0/d\tau)}\ln\left[\frac{2R_1}{r}-\frac{n_1^2\rho^2 R_1}{n_2^2\alpha^2\sigma}\left(\frac{dX_0}{d\tau}\right)^2-1\right]+\frac{R_2^2 n_2\sigma}{4R_1\eta n_1 (dX_0/d\tau)}\left[\frac{R_1}{r}-\frac{n_1^2\rho^2 R_1}{n_2^2\alpha^2 2\sigma}\left(\frac{dX_0}{d\tau}\right)^2-1\right]. \quad (24)$$

The following is the condition for the menisci in the narrow capillaries to remain in position:

$$P_0\leqslant P_{0c}=\frac{2\sigma}{R_2};\quad X_{0c}\leqslant X_0.$$

At time $\tau = 0$, the body is completely filled with liquid, so $P_0 = 0$; however, within a very short time, which is unimportant to the kinetics of the process, the narrow capillaries set up a pressure $P_0(\tau = 0) = 2\sigma/R_1$, and therefore we can determine the position of the menisci in the large capillaries that corresponds to the onset of descent of the menisci in the narrow capillaries from (24) by putting $P_0 = 2\sigma/R_2$; we get

$$L-X_{0c}=\frac{\rho}{\eta R_1\alpha\left[2\sigma\left(\frac{1}{r}-\frac{1}{R_2}\right)\right]^{0.5}}\left\{\frac{2An_1}{3n_2}\ln\left(\frac{2R_1}{R_2}-1\right)+\frac{R_2^2\sigma}{4}\left(\frac{R_1}{R_2}-1\right)\right\}. \quad (25)$$

When $X_0 < X_{0c}$, the menisci in the narrow capillaries begin to recede into the body, which corresponds to the second stage.

We introduce the mean liquid content, which corresponds to a degree of filling of the pore space

$$w=n_2+n_1\frac{X_0}{L}, \quad (26)$$

which allows us to convert from the i(X) relation to an i(w) equation. We perform this conversion in (24) to get the equation for the rate of elimination of the liquid:

$$w=w_0-\frac{2n_1^2 A}{3\eta R_1 L^2 (dw/d\tau)}\ln\left[\frac{2R_1}{r}-\frac{\rho^2 R_1 L^2}{\alpha^2 n_2^2\sigma}\left(\frac{dw}{d\tau}\right)^2-1\right]-$$
$$-\frac{R_2^2 n_2\sigma n_1}{4\eta R_1 L^2 (dw/d\tau)}\left[\frac{R_1}{r}-\frac{\rho^2 R_1 L^2}{\alpha^2 n_2^2 2\sigma}\left(\frac{dw}{d\tau}\right)^2-1\right]. \quad (27)$$

We can convert to the liquid content in (27) by multiplying both sides by the density ratio for the liquid and porous body, $\rho/(-n+1)\rho_M$.

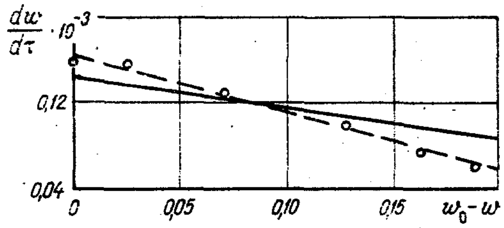

Fig. 3. Comparison of theoretical and observed rates of elimination of liquid; $dw/d\tau \cdot 10^{-3}$, sec^{-1}.

Figure 3 shows results for $dw/d\tau$ as a function of $(w_0 - w)$ derived from (27) (solid line); for comparison, we give measurements for steatite ceramic. The calculations were performed with the following parameters: $R_1 = 10^{-3}$ cm; $R_2 = 0.5 \cdot 10^{-5}$ cm; $r = 10^{-6}$ cm; $\sigma = 72$ dyn/cm; $\rho = 900$ kg/m^3; $A = 10^{-13}$ erg; $T = 333$°K, $n = 0.5$; $n_1 = n_2 = 0.25$; $n_p = 0.4$; $\eta = 0.45$ Poise; $d_p = 0.01$ cm; $L = 0.7$ cm.

The following conditions were used in calculating α. We assume that the surface roughness of the porous body is of order $R_1 = 10\ \mu$; further, $d_p = 100\ \mu > R_1$, so the area of contact of a particle with the surface is $S \approx (d_p/2)^2$. The number $\bar{n}$ of particles arriving in unit time at the area of the surface is proportional to the rms pulsation speed and the numerical particle concentration:

$$\bar{n} = \frac{3(1-\varepsilon)}{\pi d_p^3} \sqrt{\langle v^2 \rangle}. \tag{28}$$

As (11) contains f_0, which characterizes the periodicity of the contact between the fluidized bed and the surface, we have to estimate f_0 as in [12], for example. Numerical values have been estimated for the pulsation speed of the particles and the time of contact for small Reynolds numbers (Re < 1) in accordance with published recommendations [13].

Substitution in the expression for α then gives the value $2.28 \cdot 10^{-7}$ kg/m·sec·$N^{0.5}$. The agreement between theory and experiment is satisfactory in view of the very approximate determination of some of the parameters. The calculations show (Fig. 3) that the rate of removal of the liquid is almost linearly related to the liquid content of the porous body. Liquid removal from a porous system by porous particles does not show a constant-rate period, in contrast to convective drying. The rate begins to fall right from the start on account of the increased surface potential of the porous body. Note that these calculations do not incorporate any possible variation in the liquid content of the particles.

The second stage of the process starts when $X_0 = X_{0C}$, namely, the menisci in the narrow capillaries start to sink; the rate of the process should then be much less than that in the first stage, since the liquid is then removed only from the thin films on the surface. Discussion of the second stage is thus an independent problem, which will be dealt with in Part 2.

NOTATION

d_p	is the diameter of porous particles;
ρ, σ	are the density and surface tension of liquid;
η	is the viscosity;
F_1 and F_2	are the total cross-sectional areas of large and small capillaries;
$n = n_1 + n_2$	is the porosity of body;
n_p	is the porosity of particles;
Π	is the wedging pressure;
w	is the bulk liquid content;
X	is the meniscus coordinate;
L	is the characteristic dimension;
ρ_M	is the density of material;
S	is the area of contact with body;
τ	is the time;
f_0	is the fraction of time of contact with gas bubbles;
ε	is the void fraction;
$\langle \nu^2 \rangle$	is the mean-square velocity;

l is the penetration depth;
V $= dl/d\tau$;
r is the capillary radius;
$\bar{n}$ is the number of particles striking unit surface per unit time;
A is the Hamacker constant;
h is the film thickness;
N is the number of pores per unit surface of particles;
m is the mass of liquid in particle capillary;
M is the mass of liquid in a particle.

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